This week, we will take up some questions on co-ordinate geometry. Let me re-cap the relations we discussed in the last post.

Say, the equations of 2 lines are:

$ax + by + c = 0$

and

$mx + ny + p = 0$

1. A single point of intersection between two lines: $a/m \neq b/n$
2. Distinct parallel lines: $a/m = b/n \neq c/p$
3. The same line: $a/m = b/n = c/p$
4. Perpendicular lines: $am = -bn$

Question 1: A given line L has an equation $3x+4y=5$. Which of the following is the equation of line which does not intersect the above line?

(A) $4x + 3y = 5$
(B) $3x + 4y = 10$
(C) $3x + 5y = 5$
(D) $3x + 5y = 3$
(E) $3x – 4y = 5$

Solution: A line that does not intersect with L, is a distinct line parallel to L. The relation of coefficients between distinct parallel lines is $a/m = b/n \neq c/p$

Equation of L is $3x + 4y – 5 = 0$.

$a = 3$

$b = 4$

$c = -5$

For option (B), $m = 3$, $n = 4$ and $p = -10$

We see that a/m $(3/3) = b/n$ $(4/4) \neq c/p$ $(-5/-10)$

Therefore, answer is option (B).

Question 2: What is the shortest distance between the following 2 lines: $x + y = 3$ and $2x + 2y = 8$?

(A) 0
(B) 1/4
(C) 1/2
(D) $\sqrt{2}/2$
(E) $\sqrt{2}/4$

Solution:

Any two lines in the xy plane will be either parallel or intersecting. If the lines intersect, the shortest distance between them will be 0.

The two given lines are:

$x + y = 3$ (shown by the red line)
$2x + 2y = 8$ which is same as $x + y = 4$ (shown by the blue line)

We notice that a/m (1/1) = b/n (1/1) ≠ c/p (3/4). Hence, the lines are parallel.
They intersect the x axis at x = 3 and x = 4 and the y axis at y = 3 and y = 4 (as shown in the figure)

Now there are many ways of getting the distance between them. The first method I will discuss is using a formula. The second method will use the properties of right triangles.

Would I advise you to learn up the formula? No. GMAT requires you to know only the very basic formulas. This is certainly not one of them. Remember it only if you have already come across it sometime during the course of your study and seeing it here is enough for you to recall it during the exam (if need be). If you are seeing this formula for the first time, don't worry about adding it to your list. There will always be other, more intuitive ways of getting to the answer.

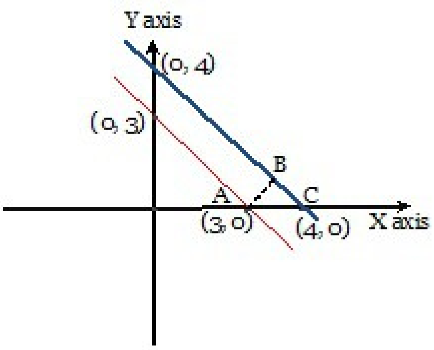
First Method: Using the formula

If the equations of two parallel lines are: $y = mx + b$ and $y = mx + c$ (note that they have the same slope, m, but different y intercepts, b and c)

Distance between them = $|b-c|/\sqrt{(m^2 + 1)}$
Here the parallel lines are: $y = -x + 3$ and $y = -x + 4$

Distance between them = $|4-3|/\sqrt{((-1)^2 + 1)} = 1/\sqrt{2} = \sqrt{2}/2$

Second Method: Using right triangles



Use the little triangle ABC. Co-ordinates of A are (3, 0) and of C are (4, 0).

When you draw the red line, you notice that its x and y intercepts are the same.
i.e. $x + y = 3$ intersects x axis at 3 and y axis also at 3. So it forms an isosceles triangle.
Similarly, $x + y = 4$ intersects x axis at 4 and y axis also at 4. It also forms an isosceles triangle so angle BCA is 45 degrees.
AB is dropped perpendicular to the blue line. This is the distance between the two parallel lines. Since angle ABC is 90 degrees, angle BAC will also be 45 degrees (to make the sum 180). So AB = AC.

In an isosceles right triangle, the ratio of the sides is 1:1: √2 where √2 is the hypotenuse. Since we know that the hypotenuse is actually 1 (= 4 – 3), the measure of equal sides (AB and BC) will be 1/√2 each. Multiply and divide this by √2 to get √2/2.

Hence, the distance between the two lines, AB = √2/2.

Answer (D).

Hope the application of the concept discussed is clear. We will continue working on co-ordinate geometry next week. Till then, keep practicing!